UNIT 1: QUANTUM MECHANICS

1 **Calculate the number of photons emitted per second by a 100 watt sodium vapour lamp. Assume λ = 5893Å for sodium light.**

Solution:
Given data:
No. of photons /second = ? λ = 5893Å

WKT $E = h\upsilon = hc/\lambda = (6.634x10^{-34}x3x10^{8})/5893x10^{-10} = 3.375x10^{-19}$ J
Further, the rate of energy emission from a 100watt lamp is 100 joules/second. So the number of photons (each having energy $3.375x10^{-19}$ emitted per second is
$100 / 3.375x10^{-19}$ = **2.96×10^{20} photons per second**

2 **In an expt. Tungsten cathode which has threshold of 2300Å is irradiated by UV light of wavelength 1800Å. Calculate i) maximum energy of emitted photoelectrons ii) work function for tungsten. Given, $h = 6.634x10^{-34}$ J-s, $1eV = 1.6x10^{-19}$J and $c = 3x10^{8}$m/s.**
Solution:
WKT
i) $KE_{max} = h(\upsilon - \upsilon_o) = h(c/\lambda - c/\lambda_o)$
$= 6.634x10^{-34}x3x10^{8} (1/1800x10^{-10} - 1/2300 x10^{-10})$ = **1.48eV**

ii) Work function $W_o = h\upsilon_o = (6.634x10^{-34}x3x10^{8} /2300 x10^{-10})$ = **5.38eV**

3 **Calculate the de Broglie wavelength of the following and justify your answers**
i. **a 1000kg automobile traveling at 100m/s**
ii. **a smoke particle of mass 10^{-9}g moving at 1cm/s**
iii. **an electron with KE 1eV**

Solution:
i. we have $\lambda = h/mv = 6.634x10^{-34}/1000x100$ = **$6.634x10^{-29}$ m**
ii. we have $\lambda = h/mv = 6.634x10^{-34}/10^{-12}x10^{-2}$ = **$6.634x10^{-20}$ m**
iii. we have $\lambda = h/(2meV)^{½} = 6.634x10^{-34}/(2x9.1x10^{-31}x1.6x10^{-19})^{½}$ = **$1.22x10^{-9}$ m**

From the above results it is clear that smaller the mass of the object, more prominent will be its de Broglie wavelength and vice versa.

4 **Calculate the de Broglie wavelength associated with a proton moving with $1/20^{th}$ the velocity of light, mass of proton is 1836 times the mass of electron.**
Solution:
$m_p = 1836\ m_e$; $m_e = 9.1x10^{-31}$kg ; v = 1/20 x velocity of light
$\lambda = h/mv = 6.634x10^{-34} \times 20/(1836x9.1x10^{-31}x3x10^{8}) = 2.64x10^{-14}$m

5 **If an electron has a de Broglie wavelength of 2nm, find its KE and group velocity.**
Solution:
$KE = p^2/2m = h^2/ 2m\lambda^2 = (6.634x10^{-34})^2/(2x9.1x10^{-31}x(2x10^{-9})^2)$ = **$6.038x10^{-20}$ J**
Group velocity = $p/m = h/m\lambda = 6.634x10^{-34}/9.1x10^{-31}x2x10^{-9}$ = **364285 m/s**

6 **An electron has a speed of 600m/s with an accuracy of 0.005%. Calculate the certainty with which the position of the electron can be located?**

Solution:

We have Heisenberg's Uncertainty principle

$\Delta x\ \Delta p \geq h/4\pi$ or $\Delta x\ m\Delta v \geq h/4\pi$

Δv is 0.005% of 600m/s that is $\Delta v = (0.005x600)/100 =$ **0.030m/s**

$\Delta x = h/4\pi\ m\Delta v = 6.63x10^{-34}/4x3.14x9.1x10^{-31}x0.03 =$ **$1.93x10^{-3}$m**

Hence, certainly there exists an electron with in **$1.93x10^{-3}$m**

7 **If an electron has a deBroglie wavelength of 2nm, find its kinetic energy and group velocity.**

Solution:

$v_g = v_{particle} = p/m = h/\lambda m = 6.63x10^{-34}/2x10^{-9}x9.1x10^{-31} =$ **$3.64\ x10^5$m/s**

8 **A particle of mass $1.67x10^{-27}$kg is confined to the 2^{nd} excited state in a one dimensional potential well of infinite height & width L = 0.1nm. Calculate (a) its energy (b) momentum and (c) the probability of finding the particle between 0 & L/3.**

Solution:

$E = n^2h^2/8ml^2 =$ **$29.34x10^{-21}$J**

$p = (2mE)^{1/2} =$ **10^{-23}kg-m/s**

$$p = \int_0^{L/3} |\psi|^2\, dx = \int_0^{L/3} \left[\sqrt{\frac{2}{L}}\ \sin\left(\frac{n\pi}{L}\right)x\right]^2 dx = \int_0^{L/3} \left[\frac{2}{L}\ \sin^2\left(\frac{n\pi}{L}\right)x\right] dx$$

$$p = \left(\frac{2}{L}\right)\left(\frac{1}{2}\right)\int_0^{L/3} (1 - \cos\left(\frac{2n\pi}{L}\right)x\ )dx = \left(\frac{1}{L}\right)\left[\int_0^{L/3} 1dx - \int_0^{L/3} \cos\left(\frac{2n\pi}{L}\right)x\ dx\right]$$

$$p = \left(\frac{1}{L}\right)\left[\frac{L}{3}\right] = \frac{1}{3}$$

9 **A nucleon is confined to a nucleus of diameter $5x10^{-4}$m. Calculate the minimum uncertainty in the momentum of the nucleon. Also calculate the minimum kinetic energy of the nucleon.**

Solution:

$\Delta x\ \Delta p \geq h/4\pi$ or $\Delta p = h/\Delta x 4\pi = 6.634x10^{-34}/5x10^{-4}x4x3.14 =$ **$1.056x10^{-31}$kg-m/s**

Since $p \geq (\Delta p)_{min}$, let us consider $p = (\Delta p)_{min}$

$E_{min} = p^2/2m_n =$ **$3.33x10^{-36}$J**

10 **If the uncertainty in the location of a particle is equal to its de-Broglie wavelength, what is the uncertainty associated with the velocity?**

Solution:

$\Delta x\ \Delta p \geq h/4\pi$ or $\Delta x\ m\Delta v \geq h/4\pi$ or $\Delta v \geq h/\Delta x\ m\ 4\pi$

But $\Delta x = \lambda$ therefore $\Delta v \sim h/(h/p)\ m\ 4\pi \sim v/4\pi$

11 **For a particle having energy E & momentum P, show that the group velocity of matter waves associated with a moving particle on which no forces act is twice the phase velocity.**

Solution: $v_{ph} = \omega/k = 2\pi\nu/(2\pi/\lambda) = \nu\lambda = (E/h)(h/p) = E/p = p^2/2mp = p/2m = 1/2(v_{particle}) = 1/2(v_{pgroup})$

Hence $v_{pgroup} = 2v_{ph}$

12 **For surface tension waves in shallow water, the relation between frequency & wavelength is given by** $\nu = \left(\frac{2\pi T}{\rho\lambda^3}\right)^{1/2}$ **find the group velocity.**

Solution: $\nu = \left(\frac{2\pi T}{\rho}\right)^{1/2}\lambda^{-3/2} = \mathrm{X}\,\lambda^{-3/2} = \mathrm{X}\left(\frac{2\pi}{k}\right)^{-3/2}$ or $\frac{\omega}{2\pi} = \nu = X(2\pi)^{-3/2}\,k^{3/2}$

$\omega = X(2\pi)^{-1/2}\,k^{3/2}$ or $\frac{d\omega}{dk} = X(2\pi)^{-1/2}\,\frac{3}{2}k^{1/2} = \frac{3}{2}X(2\pi)^{-1/2}\left(\frac{2\pi}{\lambda}\right)^{1/2} = \frac{3}{2}\left(\frac{2\pi\mathrm{T}}{\rho}\right)^{1/2}\left(\frac{1}{\lambda}\right)^{1/2} = \frac{3}{2}\left(\frac{2\pi\mathrm{T}}{\rho\lambda}\right)^{1/2}$

13 **In a particular substance the phase velocity of waves doubles when the wavelength is halved. Show that wave groups in this system move at twice the central phase velocity.**

Solution:

$v_p \propto 1/\lambda$ or $v_p = A/\lambda$, we have $v_g = v_p - \lambda[dv_p/d\lambda]$

$v_g = v_p - \lambda[-A/\lambda^2] = v_p + A/\lambda = 2\,v_p$

14 **A particle is confined to a one-dimensional box of length 0.2nm. It is found that when the energy of the particle is 230eV, its wave function has five antinodes. Find the mass of the particle and show that it can never have energy equal to 1keV.**

Solution: Given data,

L = 0.2nm, E = 230eV and for five antinodes n=5

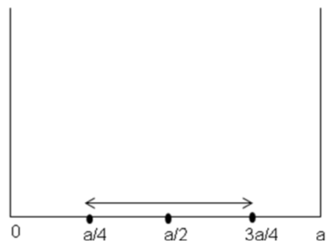
$E = n^2h^2/8mL^2$ or $m = n^2h^2/8EL^2 = 5^2 \mathrm{x}\ (6.6\mathrm{x}10^{-34})^2/8\mathrm{x}230\mathrm{x}1.6\mathrm{x}10^{-19}\mathrm{x}(0.2\mathrm{x}10^{-9})^2 =$ $\mathbf{9.24x10^{-31}kg}$

For E = 1keV we get **n =10.42** which is a fraction and hence not allowed

15 **A quantum particle confined to one-dimensional box of width 'a' is in its first excited state. What is the probability of finding the particle over an interval of a/2 marked symmetrically at the centre of the box.**

Solution:

$$p = \int_{a/4}^{3a/4} |\psi|^2\,dx = \int_{a/4}^{3a/4}\left[\sqrt{\frac{2}{\mathrm{a}}}\ \sin\left(\frac{2\pi}{\mathrm{a}}\right)x\right]^2 dx = \int_{a/4}^{3a/4}\left[\frac{2}{\mathrm{a}}\ \sin^2\left(\frac{2\pi}{\mathrm{a}}\right)x\right]\mathrm{dx} = 0.5$$



16 **A plane wave is travelling in a dispersive medium with phase velocity v_{ph} given by v_{ph} = a+bλ where a & b are constants. Show that the group velocity is equal to 'a'.**

Solution:

We have $v_{ph} = \omega/k$ ------ (1) and $v_{ph} = a+b\lambda$ ------ (2)

Combining equations (1) & (2) we get

$\omega = (a+b\lambda)k$

$$\frac{\mathrm{d}\omega}{\mathrm{dk}} = (\mathrm{a}+\mathrm{b}\lambda) + \mathrm{k}\left(0 + \mathrm{b}\frac{\mathrm{d}\lambda}{\mathrm{dk}}\right)\ \ or\ \ \frac{\mathrm{d}\omega}{\mathrm{dk}} = \mathrm{v_g} = \mathrm{a} + \mathrm{b}\left(\frac{2\pi}{\mathrm{k}}\right) + \mathrm{kb}\left(\frac{-2\pi}{\mathrm{k}^2}\right) = \mathrm{a}$$

17 **Find the value of 'A' in terms of 'k' for a wave function ψ = Aexp (-kx) for 0 < x < ∞.**

Solution :

$\int_0^\infty |\psi|^2\,\mathrm{dx} = 1$ implies $\int_0^\infty A^2\,e^{-2kx}\,\mathrm{dx} = 1$

$\frac{A^2}{-2k}\left[\frac{1}{e^\infty} - 1\right] = 1$ implies $\mathbf{A = (2k)^{1/2}}$

18 **An excited atom has an average life time of 10^{-8}s. During this period, it emits a photon and returns to the ground state. What is the minimum uncertainty in the frequency of**

this photon.

Solution:

$\Delta E\ \Delta t \geq h/4\pi$ or $h\Delta\upsilon\ \Delta t \geq h/4\pi$ or $\Delta\upsilon \geq [1/4\pi\ \Delta t]$ or $\boldsymbol{\Delta\upsilon \sim 8 \times 10^6}$ **cps**

But $\Delta x = \lambda$ therefore $\Delta v \sim h/(h/p)\ m\ 4\pi \sim v/4\pi$

19 **Estimate the minimum time spent by an atom in the excited state during the excitation & de-excitation process, when a spectral line of wavelength 546nm & width 10^{-14} m is emitted.**

Solution :

We have $E = hc/\lambda$

$\Delta E = [hc\Delta\lambda/\lambda^2]$ -------(1)

Also $\Delta E\ \Delta t \geq h/4\pi$ ------(2) Comparing eqns. (1) & (2) we get

$\Delta t \sim (\lambda^2/4\pi)(1/c\Delta\lambda) \sim (546 \times 10^{-9})^2/4\pi \times 1/3 \times 10^8 \times 10^{-14} = \mathbf{8 \times 10^{-9} s}$

20 **Compare the kinetic energy of a photon with that of an electron when both are associated with a wavelength 0.2nm.**

Solution:

$KE_{electron} = p^2/2m = h^2/2m\ \lambda^2$; $KE_{photon} = hc/\lambda$

$[KE_{electron}]/\ [KE_{photon}] = [hc/\lambda] \times [2m\ \lambda^2\ /h^2] = 2mc\lambda\ /h = [2 \times 9.1 \times 10^{-31} \times 3 \times 10^8 \times 0.2 \times 10^{-9}]\ /6.63 \times 10^{-34}$

$[KE_{electron}]/\ [KE_{photon}] =$ **[164.70/1]**

UNIT 2: ELECTRICAL & THERMAL PROPERTIES OF MATERIALS

1 **Calculate the Fermi energy of sodium assuming that the metal has one free electron per atom. Given the density of sodium = 970kg/m^3 and atomic weight of sodium = 22.99**

Solution:

$E_F = (3/\pi)^{2/3} (h^2/8m)n^{2/3} = 3.65\text{x}10^{-19}\text{x } n^{2/3}$ eV $= 3.65\text{x}10^{-19}\text{x } (N/V)^{2/3}$ eV

$= 3.65\text{x}10^{-19}\text{x } (6.023\text{x}10^{26}/[22.99/970])^{2/3}$ eV = **3.92eV**

Note: one free electron /atom means N_A atoms have N_A electrons

$n = N_A/V$ and V = mass/density

2 **Find the average drift velocity of electrons in a copper conductor with cross sectional area of 10^{-6}m^2 carrying a current of 4A. The atomic weight of Cu is 63.6 and density is 8.9g/cm^3. N_A = 6.02x10^{23}**

Solution:

$n = N_A/V = (N_A \text{ x density})/\text{mass} = (6.02\text{x}10^{26}\text{x}8.9\text{x}10^3)/63.6 =$ **$8.424\text{x}10^{28}$**

$v_d = I/nAe = 4/(8.424\text{x}10^{28}\text{x}10^{-6}\text{x}1.6\text{x}10^{-19}) =$ **$3\text{x}10^{-4}$m/s**

3 **Find the relaxation time in a current carrying metallic wire. The metal has 5.8x10^{28} conduction electrons/m^3 and its resistivity is 1.54x10^{-8} ohm-m.**

Solution:

$\sigma = 1/\rho = ne^2\tau/m$ or $\tau = m/ne^2\rho = 9.1\text{x}10^{-31}/[5.8\text{x}10^{28} \text{ x } (1.6\text{x}10^{-19})^2 \text{ x } 1.54\text{x}10^{-8}] =$ **$3.97\text{x}10^{-14}$s**

4 **The free electron density of a conductor is 10^{25}/m^3. Find the drift velocity of the free electrons when the conductor has a current whose density is 1.12x10^6 amp/m^2 .**

Solution: Given data

$n = 10^{25}/\text{m}^3$, v_d = ?, $J = 1.12\text{x}10^6$ amp/m^2

WKT, $J = I/A = q/tA = (nAL)e/(L/v_d)\text{x}A = nev_d$ or

$v_d = J/ne = 1.12\text{x}10^6/10^{25}\text{x}1.6\text{x}10^{-19} =$ **0.7m/s**

5 **Calculate the drift velocity & thermal velocity of conduction electrons in Al at 300K when an Al wire of length 5m & resistance 0.08ohm carries a current of 15A. Given the mobility of free electrons in Al is 1.28x10^{-3} m^{2}V^{-1}s^{-1}.**

Solution:

$v_{th} = [3kT/m]^{1/2} = [3\text{x}1.38\text{x}10^{-23}\text{x}300/9.1\text{x}10^{-31}]^{1/2} =$ **$1.168\text{x}10^5$m/s**

$v_d = \mu E = \mu v/d = \mu RI/d = 1.28\text{x}10^{-3}\text{x}0.08\text{x}15/5 =$ **$3.07\text{x}10^{-4}$m/s**

6 **What is the probability that a quantum state whose energy is 0.1eV above & below Fermi energy will be occupied? Assume a sample temperature of 800K**

Solution:

0.1eV above E_F implies $E_F + 0.1 = E$ i.e $E-E_F = 0.1$eV

0.1eV below E_F implies $E_F - 0.1 = E$ i.e $E-E_F = -0.1$eV

$F(E) = 1/\{\exp[(E-E_F)/kT]+1\} =$ **0.19 or 19% above E_F and**

= 0.81 or 81% below Fermi level

7 **Evaluate the Fermi function for an energy kT above the Fermi level.**

Solution:

kT above E_F implies $E_F + kT = E$ i.e $E-E_F = kT$

$F(E) = 1/\{\exp[(E-E_F)/kT]+1\} = 1/\{\exp[kT/kT]+1\} = 1/(e+1) = 1/(2.718+1) =$ **0.268**

8 **Show that the occupation probability at $E = E_F+\Delta E$ is equal to the non-occupation probability at $E = E_F - \Delta E$**

$F(E) = 1/\{\exp[(E-E_F)/kT]+1\}$ at $E = E_F+\Delta E$ we have $E - E_F = \Delta E$

Then $F(E) = 1/\{\exp(\Delta E/kT)+1\}$ let $x = \exp(\Delta E/kT)$

Non-occupation probability = 1- occupation probability

$= 1- [1/(1+x)] = 1/[1+(1/x)] =1/[1+\exp(-\Delta E/kT)]$

at $E = E_F-\Delta E$ we have $E - E_F = -\Delta E$

9 **Find the temperature at which there is 1% probability that a state with an energy 0.5eV above Fermi energy will be occupied?**

Solution:

$F(E) = 1/\{\exp[(E-E_F)/kT]+1\}$ and $E = E_F+0.5$ or $E - E_F = 0.5eV = 0.5x1.6x10^{-19}J$

$1/100 = 1/\{\exp[0.5x1.6x10^{-19} /1.38x10^{-23}xT]+1\}$

T=1261K

10 **A copper rod 19 cm long and having an area of cross section of 0.785cm^2 thermally insulated is heated at one end through 100^{0}C while the other end is kept at 30^{0}C. Calculate the amount of heat which will flow in 10 minutes along the way. Thermal conductivity of copper is 380 W/m/K.**

Solution :

Given data A = 0.785cm^2 = 785x10^{-7}m^2, x=19cm=0.19m, T_2-T_1 =100-30=70^{0}C, t= 10x60=600s and K = 380 W/m/K

Q = [KA (T_2-T_1)t]/x = 380 x 785x10^{-7} x 70x600]/0.19 = **6.6x10^{3}J**

11 **The total area of the glass window pane is 0.5m^2. Calculate how much heat is conducted per hour through the glass window pane if thickness of the glass is 6mm, the temperature of the inside surface is 23^{0}C and of the outside surface is 2^{0}C. Thermal conductivity of glass is 0.1 W/m/K.**

Solution :

Given data A = 0.5m^2, x=6mm=6x10^{-3}m, T_2-T_1 =23-2=21^{0}C, t = 1hr=3600s and K = 1 W/m/K

Q = [KA (T_2-T_1)t]/x = 1 x 0.5x21x3600]/ 6x10^{-3} = **63x10^{5}J**

12 **Calculate the electrical and thermal conductivities for a metal with relaxation time 10^{-14} second at 300K. Also, calculate the Lorentz number using the above result (Density of electrons = 6x10^{28}m^{-3}).**

Solution :

Given data τ_r = 10^{-14} second, T=300K and n = 6x10^{28}m^{-3}

$\sigma = ne^2 \tau_r /m$ = [6x10^{28} x (1.6x10^{-19})^2 x 10^{-14}]/[9.1x10^{-31}] = **1.687x10^7 ohm-m**

$K = \frac{n\pi^2 k_B^2\ T\ \tau_r}{3m}$ = [6x10^{28} x (3.14)2 x (1.38x10^{-23})^{2}x 300 x 10^{-14}]/3x9.1x10^{-31} = **124 W/m/K**

L = K/ σT = 124/(1.687x10^7 x 300) = **2.447 x 10^{-8} WΩK^{-2}**

UNIT 3: MATERIALS SCIENCE

1 **Calculate the electronic polarizability of an isolated Se atom. The atomic radius of a Se atom is 0.12nm.**

Solution:
Atomic radius of Se atom = 0.12nm
electronic polarizability $\alpha_e = 4\pi\varepsilon_o R^3 = 4 \text{x} 3.14 \text{x} 8.854 \text{x} 10^{-12} \text{x} (0.12 \text{x} 10^{-9})^3 = \mathbf{1.9226x10^{-40}\ Fm^2}$

2 **Argon gas contains $2.7x10^{25}$ atoms per m^3 at 0^oC and at 1 atmospheric pressure. Calculate the dielectric constant of the Ar gas at this temperature if the diameter of the Ar atom is 0.384nm.**

Solution:
Electronic polarizability $\alpha_e = 4\pi\varepsilon_o R^3 = 4 \text{x} 3.14 \text{x} 8.854 \text{x} 10^{-12} \text{x} (0.384 \text{x} 10^{-9})^3 = \mathbf{0.63x10^{-40} F/m^2}$
From Claussius-Mossotti relation we have
$\frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} = \frac{N\alpha}{3\epsilon_o}$ after substitution we get $\epsilon_r = \mathbf{1.0001921}$

3 **A solid contains $5x10^{28}$ identical atoms per m^3, each with a polarizability of $2x10^{-40}$ Fm^2. Assuming that the internal field is given by the Lorentz relation, calculate the ratio of the internal field to the applied field.**

Solution:
The Lorentz relation $E_i = E + [P/3\varepsilon_o]$
The Polarization $P = N\alpha_e E_i = 5x10^{28} x2x10^{-40} xE_i = 1x10^{-11} xE_i$
$E_i = E + [1x10^{-11} xE_i /3\varepsilon_o]$
$[1 - (1x10^{-11}/3\varepsilon_o)]\ E_i = E$
$[E_i / E] = 1/[1 - (1x10^{-11}/3\varepsilon_o)] = \mathbf{1.60379}$

4 **The dielectric constant of Helium at 0^oC is 1.000074. The density of atoms is $2.7x10^{25}$ atoms per m^3. Calculate the dipole moment induced in each atom when the gas is in an electric field of $3x10^4$ V/m.**
Solution:
Electronic polarizability α_e
$\alpha_e = \frac{\epsilon_o(\epsilon_r - 1)}{N}$ =[$8.854x10^{-12}$x (0.000074)]/ $2.7x10^{25}$ **= $2.4255x10^{-41}$ Fm^2**

Dipole moment = α_e E = $2.4255x10^{-41}$ x $3x10^4$ **= $7.2767 \text{ x } 10^{-37}$ C-m**

5 **What is the polarization produced in sodium chloride by an electric field of 500V/m. Given that its relative permittivity is 6?**
Solution:
$P = \epsilon_o\ (\epsilon_r - 1)E$ = $8.854x10^{-12}$x5x 500 = $2.214x10^{-8}$ C-m^2

6 **The atomic weight and density of Sulphur are 32 and $2.08x10^3$ kg/m^3 respectively. The electronic polarizability of the atom is $3.28x10^{-40}$ Fm^2. If Sulphur solid has cubic structure, calculate its dielectric constant.**
Solution:

$$\frac{(\epsilon_r - 1)}{(\epsilon_r + 2)} = \frac{N\alpha}{3\ \epsilon_o}$$

$N = N_A/V = 6.023x10^{23}/ (32x10^{-3}/2.08x10^3) = \mathbf{3.914x10^{28}}$

$$\epsilon_r = \mathbf{3.78}$$

7 **For intrinsic gallium arsenide, the room temperature electrical conductivity is 10^{-6} /Ω-m, the electron and hole mobilities are respectively 0.85 m^2/Vs and 0.04 m^2/Vs. Compute the intrinsic carrier concentration at room temperature.**

Solution:
We know that $\sigma_i = n_i e (\mu_e + \mu_h)$ or $n_i = [\sigma_i /e (\mu_e + \mu_h)] = 10^{-6}/[1.6x10^{-19} x (0.85+0.04)] =$ **$7x10^{12}$ /m^3**

8 **The following data are given for intrinsic germanium at 300K, $n_i = 2.4x10^{19}$ /m^3, $\mu_e = 0.85$ m^2/Vs $\mu_h = 0.04$ m^2/Vs. Calculate the resistivity of the sample.**

Solution:
$\sigma_i = n_i e (\mu_e + \mu_h) = 1/\rho_i$ or $\rho_i = 1/[n_i e (\mu_e + \mu_h)] = 1/[2.4x10^{19} x\ 1.6x10^{-19} x (0.39+0.19)] =$ **0.449 Ω-m**

9 **Calculate the concentration at which donor atoms need to be added to a silicon semiconductor so that it results in an n-type semiconductor with a conductivity of $2.2x10^{-4}$ S/m and the mobility of electrons being 125 $x10^{-3}$ m^2/Vs.**

Solution:
$\sigma_e = N_e e \mu_e$ or $N_e = \sigma_e /e \mu_e = 2.2x10^{-4}/[1.6x10^{-19} x\ 125\ x10^{-3}] =$ **$1.1x10^{16}$ /m^3**

10 **The Hall coefficient of a material is $-3.68x10^{-5}$ m^3/C. What is the type of charge carriers? Also calculate the carrier concentration.**

Solution:
$R_H = 1/\rho = 1/ne$ or $n = 1/ R_H e = 1/[3.68x10^{-5} x\ 1.6x10^{-19}] =$ **$1.7x10^{23}$ /m^3**

11 **The Hall coefficient of a specimen of doped silicon is found to be $3.66x10^{-4}$$m^3$/coulomb. The resistivity of the specimen is $9.93x10^{-3}$ Ω-m. Find the mobility and density of the charge carrier, assuming single carrier conduction.**

Solution:
$R_H = 1/ne = 1/ N_h e$ or $N_h = 1/ R_H e = 1/[3.66x10^{-4}x1.602x10^{-19}] =$ **$1.7055x10^{22}$/m^3**
Since we know $\sigma_e = N_e e \mu_e$ and $\sigma_h = N_h e \mu_h$ or $\mu_h = 1/\rho_i N_h e = 1/[9.93x10^{-3}x1.7055x10^{22}x1.6x10^{-19}]$
$\mu_h = 0.041$ m^2/Vs

UNIT 4: LASERS AND OPTICAL FIBERS

1 **Calculate the ratio of**

i) Einstein coefficient and

ii) Stimulated to spontaneous emissions for a system at 350K in which radiations of wavelength 1.5μm are emitted

Solution:

$A_{21}/B_{12} = 8\pi h\nu^3/c^3 = 8\pi h/\lambda^3 = 8x3.14x6.63x10^{-34}/(1.5x10^{-6})^3 = \mathbf{4.93x10^{-15}}$

Also

Rate of stimulated emission/Rate of spontaneous emission = $B_{21}N_2E(\nu)/A_{21}N_2 = B_{21}E(\nu)/A_{21}$ -------(1)

But $E(\nu) = [8\pi h\nu^3/c^3][1/\{\exp(h\nu/kT) -1\}] = \{A_{21}/B_{21}\}[1/\{\exp(h\nu/kT) -1\}]$ -------(2)

Comparing eqns.(1) & (2) we get

[Rate of stimulated emission/Rate of spontaneous emission] = $B_{21}E(\nu)/A_{21}$

$= (B_{21}/A_{21})\ (A_{21}/B_{21})[1/\exp(h\nu/kT) -1]$

$= [1/\exp(h\nu/kT) -1] = \mathbf{1.19x10^{-12}}$

2 **A pulse from a laser with power 1mW lasts for 10ns. If the number of photons emitted in a pulse is $3.491x10^7$, calculate the wavelength of laser.**

Solution: We have

No. of photons x Energy = Power x Time

Energy = (Power x Time)/ No. of photons

$hc/\lambda = (1x10^{-3}x10x10^{-9})/3.491x10^7 = 2.86x10^{-19}$

$\lambda = hc/2.86x10^{-19}$ =**6943Å**

3 **At what temperature are the rates of spontaneous & stimulated emission equal? Given λ = 400nm.**

Solution:

[Rate of stimulated emission] / [Rate of spontaneous emission] = $B_{21}N_2E(\nu)/A_{21}N_2$

$= B_{21}E(\nu)/A_{21}$

But $E(\nu) = [8\pi h\nu^3/c^3][1/\{\exp(h\nu/kT) -1\}] = A_{21}/B_{21}[1/\{\exp(h\nu/kT) -1\}]$

[Rate of stimulated emission] / [Rate of spontaneous emission] =1

$(B_{21}/A_{21})\ (A_{21}/B_{21})[1/\exp(h\nu/kT) -1] = 1$

$\exp(h\nu/kT) -1 = 1$

$\exp(h\nu/kT) =2$ or $h\nu/kT = \ln 2$ or

$T = hc/\lambda k \ln 2 = [6.634x10^{-34}x3x10^8]/[400x10^{-9}x1.38x10^{-23}x0.6931]$=**52019K**

4 **Calculate the NA, relative refractive index difference, V-number and no. of modes in an optical fiber of core diameter 50μm, core & cladding RIs 1.41 & 1.40 at wavelength 820nm.**

Solution:

$NA = [(n_1)^2-(n_2)^2]^{½} /n_o = [(1.41)^2-(1.40)^2]^{½} = \mathbf{0.167}$

$V = (\pi d/\lambda)NA = [(3.14x50x10^{-6})/820x10^{-9}]x0.167 = \mathbf{31.97}$

No. of modes = $V^2/2$ = **511**

5 **An optical fiber has cladding of RI 1.50 & NA 0.39. Find the RI of the core and the acceptance angle**

Solution:

$NA = [(n_1)^2-(n_2)^2]^{½}$ or $(n_1)^2 = (NA)^2 + (n_2)^2 = (0.39)^2 + (1.50)^2$ or $\mathbf{n_1 = 1.54}$

Also we know that $NA = \sin\theta_a$ or $\theta_a = \sin^{-1}(0.39) =$ **22.92 degree**

6 **The attenuation of light in an optical fiber is estimated at 2.2dB/km. What fractional initial intensity remains after 8km?**

Solution:

Given, $\alpha = 2.2$dB/km, L =8km, $[P_{out}/P_{in}]$=?

$\alpha = (-10/L)\log_{10}[P_{out}/P_{in}]$dB/km

$\log_{10}[P_{out}/P_{in}] = -\alpha L/10 = -2.2x8/10 = -1.76$

$[P_{out}/P_{in}]=10^{-1.76} =$ **0.0173**

7 **Calculate the number of modes tha an optical fiber would transmit with the following data. $n_{core} = 1.50$, $n_{clad} = 1.48$, core radius = 50µm, $\lambda = 1$µm**

Solution:

$V = (\pi d/\lambda)NA = (3.14/1x10^{-6})x[(1.50)^2-(1.48)^2]^{½} = 76.65$

No. of modes = $V^2/2 = 5875$

8 **The numerical aperture of an optical fiber is 0.39. If the difference in the RIs of the material of its core & the cladding is 0.05, calculate the RI of the material of the core.**

Solution:

Given NA = 0.39, $n_1-n_2 = 0.05$, n_1=? and let n_o =1

We have $NA = [(n_1)^2-(n_2)^2]^{½}/n_o$

$[NA \times n_o]^2 = [(n_1-n_2)(n_1+n_2)] = [(0.05)(n_1+n_2)]$

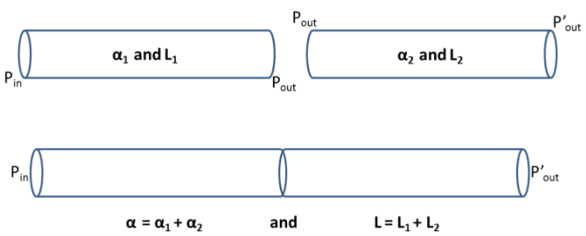
$(n_1+n_2) = [0.39 \times 1]^2/0.05 = 3.042$

$0.05+n_2+n_2 = 3.042$

$n_2 =$ **1.546** & $n_1 = 0.05+ n_2 =$ **1.596**

9 **Two optical fibers A&B of lengths L_1 & L_2 are connected in series. The output of A is fed as input of B. If the attenuation coefficients of A&B are α_1 & α_2 respectively then show that the attenuation coefficient of the combination of the optical fiber as $[\alpha_1 L_1+ \alpha_2 L_2]/[L_1+L_2]$**

Solution:



$\alpha_1 = (-10/L_1)\log_{10}[P_{out}/P_{in}]$ or $\log_{10}[P_{out}/P_{in}] = (-\alpha_1 L_1/10)$ --------(1)

$\alpha_2 = (-10/L_2)\log_{10}[P'_{out}/P_{out}]$ or $\log_{10}[P'_{out}/P_{out}] = (-\alpha_2 L_2/10)$ --------(2)

$\alpha = (-10/L_1 + L_2)\log_{10}[P'_{out}/P_{in}] = (-10/L_1 + L_2)[\log_{10}(P'_{out}/P_{out})(P_{out}/P_{in})]$

$\alpha = (-10/L_1 + L_2)[\log_{10}(P'_{out}/P_{out}) + \log_{10}(P_{out}/P_{in})]$

$\alpha = (-10/L_1 + L_2)(-\alpha_2 L_2/10 - \alpha_1 L_1/10)$

$\alpha = [\alpha_1 L_1+ \alpha_2 L_2]/[L_1+L_2]$

Unit 5 : Theory of Oscillations

1. A 3.94 kg block extends a spring 15.7 cm from its un stretched position. The block is removed and a 0.520 kg object is suspended from the same spring and is set into oscillations. Find the period of oscillation.

Solution: M=3.94 kg, x=15.7 cm

m=0.520 kg what is period?

$$Mg = kx \Rightarrow k = \frac{Mg}{x} = \frac{3.94 \times 9.8}{0.157}$$

= 245.9 N m^{-1}

Now the spring is set into oscillations with mass m=0.520 kg

The period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.520}{245.9}} = 0.298\,s$

2. A particle executes SHM of amplitude 5 cm. When the particle is 3 cm from the mean position its acceleration is found to be 48 cm s^{-2}. Calculate (a) its velocity at the same instant, (b) its time period and (c) its maximum velocity.

Solution: x_m=5 cm; x=3 cm; a=48 cm s^{-2}

(a) Velocity at position x is

$v = \pm\sqrt{\frac{k}{m}(x_m^2 - x^2}$ we need to find k/m

$ma = kx$ gives $\frac{k}{m} = \frac{a}{x} = \frac{48}{3} = 16\,s^{-2}$

$$\therefore v = \pm\sqrt{16(25-9)} = \sqrt{16 \times 16} = 16\,cm\,s^{-1}$$

(b) Time period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1}{16}} = 1.57\,s$

(c) Maximum velocity occurs at x=0

$$v_{max} = \sqrt{\frac{k}{m}x_m^2} = \sqrt{16 \times 25} = 20\,cm\,s^{-1}$$

3. A 5.22 kg object is attached to the bottom of a spring and set vibrating. The maximum speed of the object is 15.3 cm s^{-1}, and the period is 645 ms. Find (a) the force constant, (b) the amplitude of the motion and (c) the frequency of oscillation.

Solution: m=5.22 kg, v_{max}=15.3 cm s^{-1}, T=0.645 s

(c) $f = \frac{1}{T} = \frac{1}{0.645} = 1.55\,\text{Hz}$

(a) $T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow k = \frac{4\pi^2 m}{T^2}$

$= \frac{4\pi^2 \times 5.22}{0.645^2} = 495.3\,\text{N m}^{-1}$

(b)

$v_{max} = \pm\sqrt{\frac{k}{m} x_m^2} \Rightarrow x_m = \frac{v_{max}}{\sqrt{\frac{k}{m}}}$

$= \frac{0.153}{94.89} = 1.57$ cm

4. A certain spring hangs vertically. When a body of mass M=1.65 kg is suspended from it, its length increases by 7.33 cm. The spring is then mounted horizontally, and a block of mass m=2.43 kg is attached to the spring. The block is free to slide along a frictionless horizontal surface. Calculate (a) the force constant of the spring, (b) how much horizontal force is required to stretch the spring by a distance of 11.6 cm? and (c) when the block is displaced a distance of 11.6 cm and released, with what period it will oscillate?

Solution: M=1.65 kg, x=0.0733 m

M=2.43 kg, x=0.116 m

(a) At equilibrium,

$kx = Mg \Rightarrow k = \frac{Mg}{x}$

$= \frac{1.65 \times 9.8}{0.0733}\ \text{N m}^{-1}$

(b) Using Hooke's law

$F = kx = 221 \times 0.116 = 25.6\ \text{N}$

(c) Period

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2.43}{221}} = 0.6589 \text{ s}$$

=659 ms

5. The vibration frequencies of atoms in a solid are of the order of 10 THz at normal temperatures. Imagine the atoms to be connected to one another by springs. Suppose that a single silver atom vibrates with this frequency and all other atoms are at rest. Compute the effective force constant. One mole of silver has a mass of 108 g and contains 6.02×10^{23} atoms.

Solution: Mass of N_A silver atoms=108×10^{-3} kg

Mass of one atom of silver is

$$m = \frac{108\times10^{-3}}{6.02\times10^{23}} \text{ kg}$$

The angular frequency

$$\omega = 2\pi f = 2\pi\times10^{12} = 6.2813\times10^{13} \text{ rad s}^{-1}$$

$$\omega^2 = \frac{k}{m} \text{ gives } k=\omega^2 m$$

$$\text{i. e., } k=(6.2813\times10^{13})^2 \times \frac{108\times10^{-3}}{6.02\times10^{23}}$$

$$= 708.2 \text{ N m}^{-1}$$

6. An automobile can be considered to be mounted on four springs as far as vertical oscillations are concerned. The springs of a certain car of mass 1460 kg are adjusted so that the vibrations have frequency of 2.95 Hz. (a) Find the force constant of each spring (assumed identical) and (b) what will be the vibration frequency if five persons, averaging 73.2 kg each ride the car?

Solution: The total mass=1460 kg. This load is equally divided among the four springs. Therefore, load on each spring is $m = \frac{1460}{4} = 365$ kg

$$\omega^2 = \frac{k}{m} \text{ gives } k=m\omega^2 = (2\pi)^2 m f^2$$

$$k = (2\pi)^2 \times 365 \times 2.95^2 = 125399.7 \text{ N m}^{-1}$$

Now, when five persons are riding the car, the load on the springs increase.

The total load is =1460+73.2×5=1829 kg

Now the effective load on each spring is

$$m = \frac{1826}{4} 456.5 \text{ kg}.$$

Now,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{125399.7}{465.5}} = 16.57 \text{ rad s}^{-1}$$

$$f = \frac{\omega}{2\pi} = \frac{16.57}{2\pi} = 2.63 \text{ Hz}$$

7. In a thought experiment a tunnel is drilled through the Earth from pole to pole. A body of mass 10 kg is dropped at one pole. The body executes SHM. Find (a) its period, (b) force acting on the body, and (c) the velocity of the body when it crosses the center of the Earth.

Given: Radius of the Earth R_E=6000 km

Solution: We have

$$mg = kx \Rightarrow \frac{k}{m} = \frac{g}{x} = \frac{9.8}{6000 \times 10^3}$$

$$\frac{k}{m} = 1.633 \times 10^{-6} \text{ s}^{-2}$$

$$\frac{m}{k} = 6.122 \times 10^5 \text{ s}^2$$

(a) Period,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{6.122 \times 10^5} = 4916 \text{ s}$$

=1 h 21 m 57 s

(b) Force,

$$F = mg = 10 \times 9.8 = 98 \text{ N}$$

(c) Maximum velocity occurs when the particle crosses the center of the Earth.

$$v_{\text{at the center}} = \pm\sqrt{\frac{k}{m} x_m^2 - x^2}$$

$$v = \pm\sqrt{1.633 \times 10^{-6} \times \left[(6000 \times 1000)^2 - 0^2\right]}$$

$$= \pm 7667 \text{ m s}^{-1}$$

8. The amplitude of an oscillator of frequency 200 cycles per second falls to 1/10 of its initial value after 2000 cycles. Calculate (a) its relaxation time, (b) quality factor, (c) time during which the energy falls to 1/10 of its initial value, and (d) the damping constant.

Solution: The instantaneous amplitude of a damped oscillator is

$$x = Pe^{-bt}$$

(d) Let $x=x_0$ at t=0 s be the initial amplitude.

Since the frequency is 200 cycles/second, 2000 cycles correspond to 10 s.

Therefore, after 10 s, the amplitude is $x_0/10$.

$$\frac{x_0}{10} = x_0 e^{-b\times 10} \Rightarrow 10 = e^{10b}$$
$$\ln 10 = 10b$$
$$\therefore b = \frac{\ln 10}{10} = 0.23 \text{ s}^{-1}$$

(a) Relaxation time,

$$\tau = \frac{1}{2b} = \frac{1}{2\times 0.23} = 2.172 \text{ s}$$

(b) Quality factor,

$$Q = 2\pi f\tau = 2\pi\times 200\times 2.174 = 2730$$

$$E = E_0 e^{-\frac{t}{\tau}} \text{ gives } \frac{E_0}{10} = E_0 e^{-\frac{t}{\tau}}$$

(c) $\therefore e^{\frac{t}{\tau}} = 10 \Rightarrow \ln 10 = \frac{t}{\tau}$

$$t=\tau\ln 10=2.174\times 2.3=5 \text{ s}$$

9. The Q factor of a spring loaded with 0.3 kg is 60. It vibrates with a frequency of 2 Hz. Calculate the force constant and the mechanical resistance.

Solution: Frequency of an un damped oscillator is $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

Therefore,

$$f^2 = \frac{1}{(2\pi)^2}\frac{k}{m} \text{ gives } k=4\pi^2 f^2 m$$
$$k = 4\times\pi^2\times 4\times 0.3 = 47.37 \text{ N m}^{-1}$$

The quality factor,

$$Q = \omega\tau = \frac{\omega}{2b} = \frac{\omega}{rm}$$
$$r = \frac{\omega m}{Q} = \frac{2\pi f m}{Q} = \frac{2\times\pi\times 2\times 0.3}{60}$$
$$= 0.06282 \text{ kg m}^{-1}$$

10. An oscillator starts with an initial amplitude of 5 cm, with period, T=0.897 s. The damping forces in the system with coefficient b=0.075 s^{-1} continuosly reduce the amplitude. Calculate the logarithmic decrement and amplitudes after first and second periods of motion.

Given: b=0.075 s^{-1}, T=0.897 s

Solution: logarithmic decrement

$$\lambda = bT = 0.075 \times 0.899 = 0.07627$$

Ratio of successive amplitudes

$$\frac{x_0}{x_1} = e^{bT} = 1.0695$$

$$x_1 = \frac{x_0}{bT} = \frac{x_0}{1.0695} = 4.674\,\mathrm{cm}$$

$$x_2 = \frac{x_1}{bT} = \frac{x_1}{1.0695} = 4.371\,\mathrm{cm}$$